An Introduction to PALM:
Filtering – Filter methods

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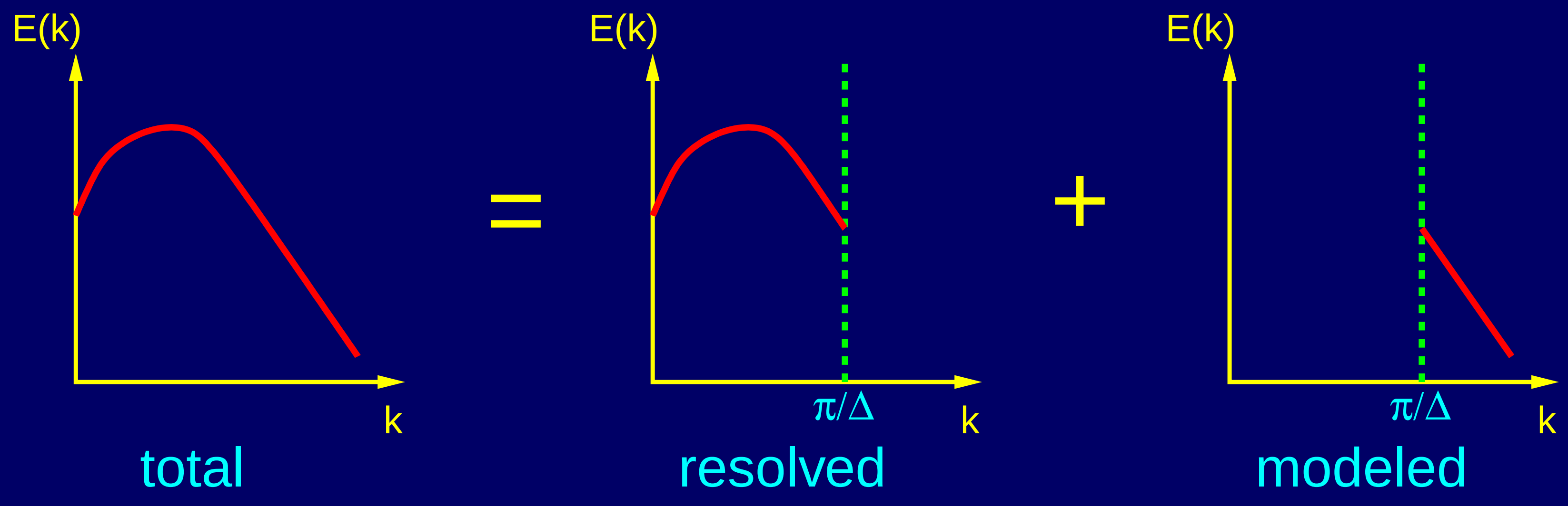
Introduction

- The large-eddy simulation technique is based on scale separation, in order to reduce number of degrees of freedom of the solution
- Low-frequency modes are calculated directly (resolved scales)
- Small/subgrid/subfilter scales are parameterized by a way of a statistical model (sub-grid/sub-filter model)
- These two categories of scales are separated by defining a cutoff length Δ



Filtered Navier-Stokes equations

- Applying the filter to the Navier-Stokes equations leads to

Momentum equations:

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\overline{u_j u_i}) = -\frac{1}{\varrho_0}\frac{\partial \overline{p}^*}{\partial x_i} + \frac{g}{\theta_{v00}}(\overline{\theta}_v - \theta_{v00})\delta_{i3} \, .$$

Equation of continuity:

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0$$

Equations for scalar variables:

$$\frac{\partial \overline{\psi}}{\partial t} = -\frac{\partial(\overline{u_i \psi})}{\partial x_i}$$

- The filtering process yields non-linear terms, e.g. $\overline{u_j u_i}$,
- In order for these equations to be usable the non-linear terms (e.g., $\overline{u_j u_i}$) have to be expressed as a function of the filtered quantities (e.g., $\overline{u}_i$) and of sub-filter scales (u_i') (decomposing the non-linear terms)

Decomposing non-linear terms – Leonard's decomposition (I)

- Introducing the basic definition of filtered and sub-filter scales the non-linear term could be expressed as a triple summation:

$$\overline{u_i u_j} = \overline{(\overline{u}_i + u_i')(\overline{u}_j + u_j')} = \overline{\overline{u}_i \overline{u}_j} + \overline{\overline{u}_i u_j'} + \overline{\overline{u}_j u_i'} + \overline{u_i' u_j'}$$

- Non-linear term is entirely written as a function of filtered and sub-filter scales ($\overline{u}_i$ and u_i')
- Two versions of introducing this decomposition to the Navier-Stokes equations exist:
  1. Double decomposition (straight forward solution, not considered here)
  2. Triple decomposition

Decomposing non-linear terms – Leonard's decomposition (II)

Triple decomposition

- Considers that it must be possible to evaluate the terms directly from the filtered variables
  - ➠ $\overline{\overline{u}_i\overline{u}_j}$ cannot be calculated directly, requires a second application of the filter
- Leonard proposes a further decomposition

$$\overline{\overline{u}_i\overline{u}_j} = (\overline{\overline{u}_i\overline{u}_j} - \overline{u}_i\overline{u}_j) + \overline{u}_i\overline{u}_j = L_{ij} + \overline{u}_i\overline{u}_j \ .$$

- Defining τ_{ij} as

$$\tau_{ij} = L_{ij} + C_{ij} + R_{ij} = \overline{u_i u_j} - \overline{u}_i\overline{u}_j$$

leads to

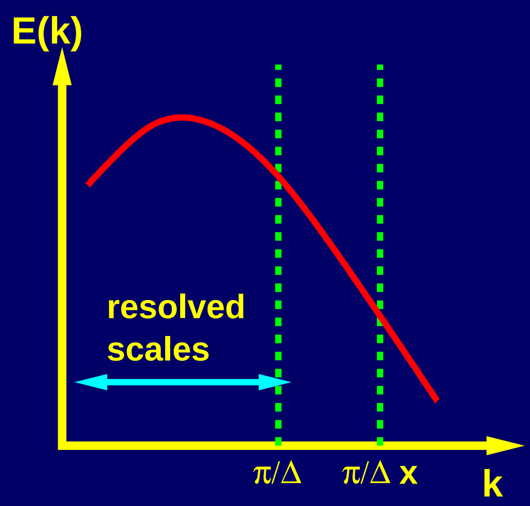
$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\overline{u}_j\overline{u}_i) = -\frac{1}{\varrho_0}\frac{\partial \overline{p}^*}{\partial x_i} + \frac{g}{\theta_{v00}}(\overline{\theta}_v - \theta_{v00})\delta_{i3} - \frac{\partial \tau_{ij}}{\partial x_j}$$

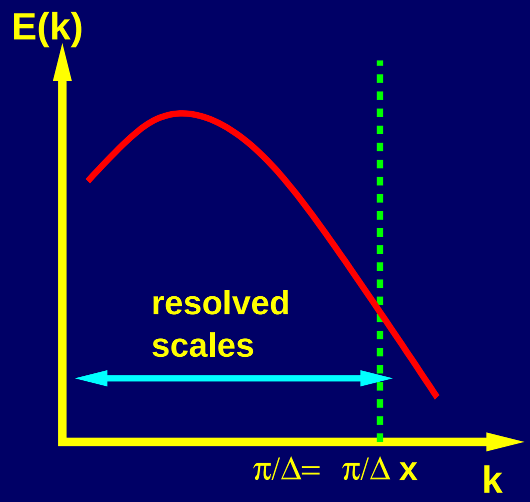
Intermediate summary

- The large-eddy simulation technique is a technique for reducing the number of degrees of freedom by separating the scales
- The selection is made by a filtering technique (low-pass filter)
- Two categories: resolved scales and sub-grid scales
- The complexity of the solution is reduced by retaining only the large scales
- The information concerning the small scales is contained in the sub-grid tensor
- In order for the dynamics of the resolved scales to remain correct, the sub-grid terms have to be considered ➪ sub-grid models

Theory and practice

- The previous developments completely ignore the computational grid used for solving the Navier-Stokes equations numerically
- The computational grid introduces another space scale:
  the discretization step Δx
- Δx has to be small enough to be able to apply the filtering process correctly: $\Delta x \leq \mathit{\Delta}$
- Two possibilities:
  ① Pre-filtering technique ($\Delta x < \mathit{\Delta}$)
  ② Linking the analytical filter to the computational grid ($\Delta x = \mathit{\Delta}$)





Theory and practice (II)

Pre-filtering:

- Increases the number of degrees in the numerical solution without increasing the number of degree of freedom in the physically resolved solution
- Requires that the analytical filter is performed explicitly
- Rarely used in practice, due to additional computational costs

Linking the analytical filter to the computational grid:

- The analytical cutoff length is associated with the grid spacing
- This method does not require the use of an analytical filter
- Brings some disadvantages concerning the evaluation of sub-grid models (should not be discussed further)
- Because of its simplicity, this method is used by nearly all authors (e.g., see Schumann, 1975)

The final set of equations:

Momentum equations:

$$\frac{\partial \overline{u}_i}{\partial t} = -\frac{\partial(\overline{u}_j \overline{u}_i)}{\partial x_j} - \frac{1}{\varrho_0} \frac{\partial \overline{p}^*}{x_i} - (\varepsilon_{ijk} f_j \overline{u}_k - \varepsilon_{i3k} f_3 u_{k_{geo}}) + \frac{g}{\theta_{v_{00}}} (\overline{\theta}_v - \theta_{v_{00}}) \delta_{i3} - \frac{\partial \tau_{ij}}{\partial x_j}$$

Equation of continuity:

$\frac{\partial \overline{u}_i}{\partial x_i} = 0$ (flow is divergence free by assumption, assured by pressure solver)

Liquid water potential temperature:

$$\frac{\partial \overline{\theta}_l}{\partial t} = -\frac{\partial(\overline{u}_i \overline{\theta}_l)}{\partial x_i} - \frac{\partial W_i}{\partial x_i} + \left(\frac{\partial \overline{\theta}_l}{\partial t}\right)_{\mathrm{rad}} + \left(\frac{\partial \overline{\theta}_l}{\partial t}\right)_{\mathrm{prec}}$$

Total water content:

$$\frac{\partial \overline{q}}{\partial t} = -\frac{\partial(\overline{u}_i \overline{q})}{\partial x_i} - \frac{\partial H_i}{\partial x_i} + \left(\frac{\partial \overline{q}}{\partial t}\right)_{\mathrm{prec}}$$

$$W_i = \overline{u_i \theta_l} - \overline{u}_i \overline{\theta}_l$$

$$H_i = \overline{u_i q} - \overline{u}_i \overline{q}$$

$$\underbrace{\tau_{ij} = \overline{u_j u_i} - \overline{u}_j \overline{u}_i}$$

parameterized

sub-grid model

Literature

Sagaut, P, 2001: Large eddy simulation for incompressible flows: An introduction. — Springer Verlag, Berlin/Heidelberg/New York, 319 p.

Schumann, U., 1975: Subgrid scale model for finite difference simulations of turbulent flows in plane channels and annuli. – J. Comp. Phys., **18**, 376–404